Wing Design for Hanggliders Having Minimum Induced Drag

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Introduction

THERE are lifting line design methods for rigid wings having minimum induced drags with practical constraints.[1-3] Recently, Wohlfahrt[4] and Nickel[5] have analyzed hanggliders with large aspect ratios and small sweep-back angles by using the lifting line theory. It is of practical interest to look for the optimum wing design taking account of a bending moment at the wing root, because the leading edge spars are the most important structural members of hanggliders. This note presents the optimum solutions.

Minimization of Induced Drag

Circulations, having a minimum induced drag, a given lift, and a given bending moment at the wing root, are known as[2,3]

$$\gamma(y) = \frac{\gamma_{Re}}{\sigma^3}\left\{(3\sigma - 2)\sqrt{1 - y^2} + 3(\sigma - 1)y^2 \ln\left|\frac{1 - \sqrt{1 - y^2}}{1 + \sqrt{1 - y^2}}\right|\right\} \quad (1)$$

where γ, γ_{Re}, and σ denote the nondimensional circulation, the circulation at the root of the elliptic wing, and the ratio of the semispan to that of the elliptic wing, respectively. The variable y is made dimensionless by using the semispan.

The ratio of D, the induced drag of the above mentioned wing, to D_e, that of the elliptic wing, is obtained as[2,3]

$$\frac{D}{D_e} = \frac{9\sigma^2 - 16\sigma + 8}{\sigma^4} = \frac{(3\sigma - 8/3)^2 + 8/9}{\sigma^4} \geqq 0 \quad (2)$$

Hence, larger σ implies smaller drag.

Mechanical Boundary Condition

Let us consider the trailing-edge deformation after Wohlfahrt[4] and Nickel.[5] Aerodynamic load and tensile forces are in equilibrium along the trailing edge (Fig. 1). Because the local center of pressure is approximately located at the $\frac{1}{4}$-chord, the mechanical boundary condition for the local lift and the tension T is given by

$$h''(y) = -(\tau/2)\gamma(y) \quad (3)$$

where h is the displacement of trailing edge made dimensionless by using the semispan, τ is the Weber number defined by

$$\tau = \tfrac{1}{2}\rho U_\infty^2 b^2/T \quad (4)$$

where ρ, U_∞, and b denote the air density, the undisturbed airflow speed, and the semispan, respectively.

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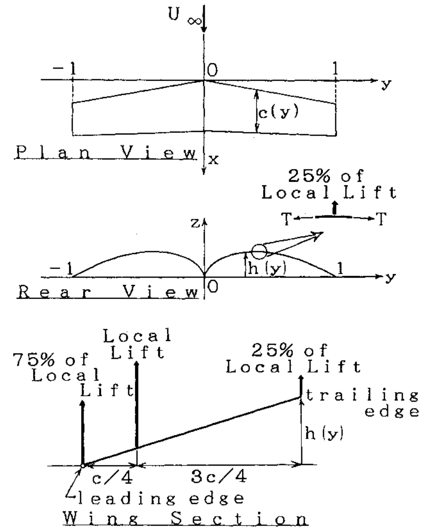

Fig. 1 Geometry of a hangglider.

Substituting Eq. (1) into Eq. (3), we can solve Eq. (3) with respect to $h(y)$ along with conditions: $h(0) = h(+1) = h(-1) = 0$

$$h(y) = \frac{\tau\gamma_{Re}}{2\sigma^3}\left[(3\sigma - 2)\left\{\frac{1}{3} + \left(\frac{\pi}{4} - \frac{1}{3}\right)|y| - \frac{1}{6}(2 + y^2)\sqrt{1 - y^2} - \frac{y}{2}\sin^{-1}y\right\} + 3(1 - \sigma)\left\{\frac{1}{3} + \left(\frac{\pi}{6} - \frac{1}{3}\right)|y| + \frac{y^4}{12}\ln\left|\frac{1 - \sqrt{1 - y^2}}{1 + \sqrt{1 - y^2}}\right| - \frac{1}{6}(2 - y^2)\sqrt{1 - y^2} - \frac{y}{3}\sin^{-1}y\right\}\right] \quad (5)$$

We must also consider the twist due to the displacement of trailing edge (Fig. 1). The theorem of Kutta-Joukowski gives

$$\gamma(y) = \tfrac{1}{2}C_{1_\alpha}c(y)\{\alpha - h(y)/c(y) - \alpha_i(y)\} \quad (6)$$

where C_{1_α}, c, α, and α_i denote the local lift curve slope assumed to be 2π, the local chord made dimensionless by using the semispan, the geometrical angle of attack, and the induced angle of attack, respectively. Solving Eq. (6) with respect to c, we obtain

$$c(y) = \{\gamma(y) + \pi h(y)\}/[\pi\{\alpha - \alpha_i(y)\}] \quad (7)$$

Let c_R denote $c(0)$, and Eq. (7) is rearranged as

$$c(y)/c(0) = \frac{\gamma(y)/\gamma(0) + \pi h(y)/\gamma(0)}{1 + \dfrac{6}{4}\pi^2\dfrac{\sigma - 1}{3\sigma - 2}c_R y} \quad (8)$$

Feasibility Study

In case of an inverse design problem, it is necessary to examine whether solutions are feasible. Solutions must satisfy the constraint on the chord: $c(y) \geqq 0$. If $c'(1)$ is positive, then $c(y)$ becomes negative in the vicinity of wing tips. Substituting Eqs. (1) and (5) into Eq. (8) and differentiating with respect to y, we have

$$\lim_{y\to 1}\frac{dc}{dy} = \frac{c(0)}{1 + \dfrac{6}{4}\pi^2\dfrac{\sigma - 1}{3\sigma - 2}c_R}\left\{\frac{3\sigma - 4}{3\sigma - 2}\lim_{y\to 1}\frac{y}{\sqrt{1 - y^2}} - \frac{\pi\tau}{6(3\sigma - 2)}\right\}\begin{cases} >0, & \text{if } \sigma > 4/3 \\ <0, & \text{if } 4/3 \geqq \sigma \geqq 1 \end{cases} \quad (9)$$

Equation (9) is confined to σ larger than unity, because larger σ implies smaller drag. According to Eq. (9), one of necessary conditions for σ is given by

$$4/3 \geqq \sigma \geqq 1 \tag{10}$$

Hence, the following inequality holds for all $y \in [0, +1]$:

$$\frac{\mathrm{d}\gamma}{\mathrm{d}y} = -\frac{(3\sigma - 2)\gamma_{Re}y}{\sigma^3\sqrt{1 - y^2}}\left[1 - \frac{6(\sigma - 1)}{3\sigma - 2}\right.$$
$$\left.\times\left\{\sqrt{1 - y^2}\,\ell n\left|\frac{1 - \sqrt{1 - y^2}}{1 + \sqrt{1 - y^2}}\right| + 1\right\}\right] \leqq 0 \tag{11}$$

Then we can deduce: 1) $\gamma(y)$ is not negative for all y because of the inequality, Eq. (11), and conditions on $\gamma(y)$; that is, $\gamma(0) > 0$ and $\gamma(1) = 0$; 2) the displacement of the trailing edge $h(y)$ is positive, as far as $\gamma(y)$ is positive (Fig. 1); and 3) Eq. (8) implies that $c(y)$ is positive, because both $\gamma(y)$ and $h(y)$ are positive.

This deduction leads to the conclusion that Eq. (10) is also the sufficient condition of positive $c(y)$. Hence, we obtain the optimum semispan ratio σ_{opt} and the optimum drag ratio

$$\sigma_{\text{opt}} = 4/3 \tag{12}$$

$$D/D_e = 27/32 \tag{13}$$

Optimum Solutions

Substituting Eq. (12) into Eq. (1), we obtain

$$\gamma(y) = \frac{27}{32}\gamma_{Re}\left\{\sqrt{1 - y^2} + \frac{1}{2}y^2\,\ell n\left|\frac{1 - \sqrt{1 - y^2}}{1 + \sqrt{1 - y^2}}\right|\right\} \tag{14}$$

Figure 2 shows the numerical result of this optimum circulation compared with the elliptic loading. Most of the necessary lift is produced near the wing root so that the bending moment would not exceed the given value.

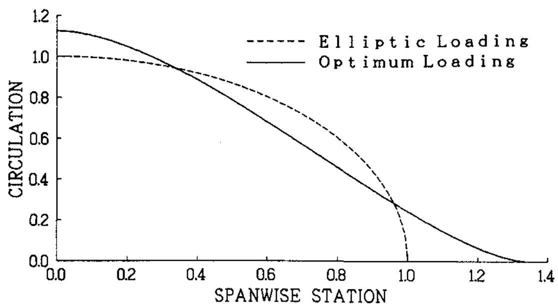

Fig. 2 Circulation distributions normalized by γ_{Re}.

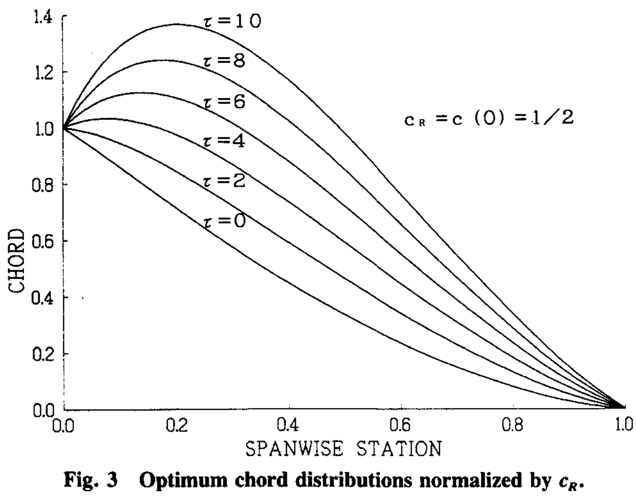

Fig. 3 Optimum chord distributions normalized by c_R.

Substitution of Eq. (12) into Eq. (8) yields

$$c(y)/c(0) = \left[\sqrt{1 - y^2} + \frac{y^2}{2}\,\ell n\left|\frac{1 - \sqrt{1 - y^2}}{1 + \sqrt{1 - y^2}}\right|\right.$$
$$+ \frac{\pi\tau}{12}\left\{1 + (\pi - 1)|y| - \left(1 + \frac{3}{2}y^2\right)\sqrt{1 - y^2}\right.$$
$$\left.\left. - \frac{y^4}{4}\,\ell n\left|\frac{1 - \sqrt{1 - y^2}}{1 + \sqrt{1 - y^2}}\right| - 2y\sin^{-1}y\right\}\right]\Big/\left\{1\right.$$
$$\left. + \frac{\pi^2}{4}c_R|y|\right\} \tag{15}$$

Figure 3 shows numerical results of Eq. (15), in the case where c_R is $\frac{1}{2}$. Hanggliders have fuller chord distributions than that of a rigid wing (i.e., $\tau = 0$). To realize the optimum circulation distribution (Fig. 2), wing area must increase wherever the effective angle of attack is reduced by the twist due to the displacement of trailing edge.

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Optimization of Constant Altitude–Constant Airspeed Flight of Turbojet Aircraft

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Nomenclature

C	=	thrust-specific fuel consumption
C_D	=	$2D/\rho V^2S$, drag coefficient, $C_D = C_{D0} + KC_L^2$
C_{D0}	=	zero-lift drag coefficient
C_L	=	$2L/\rho V^2S$, lift coefficient
D	=	drag force on aircraft
E	=	$L/D = C_L/C_D$, aerodynamic efficiency of aircraft
h	=	altitude
K	=	induced drag factor
L	=	lift force on aircraft
S	=	wing planform area
T	=	thrust required
t	=	endurance, time taken during the cruise

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